Revisiting the Simulink Library with Complementarity Conditions

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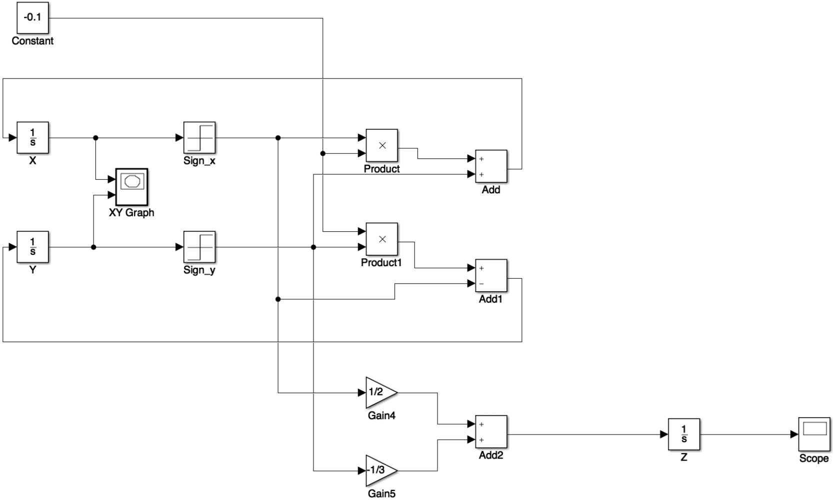
A square helix

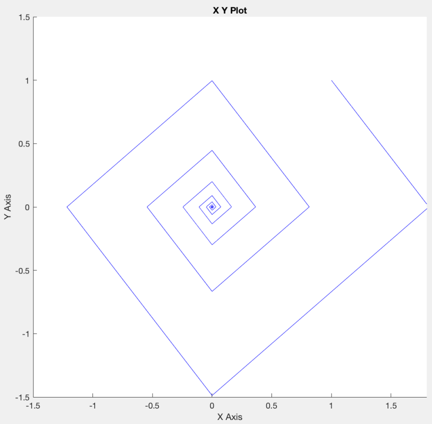
From Complementarity Conditions to Nonsmooth Dynamical Systems

Infusing Complementarity Conditions in the Standard Simulink Library

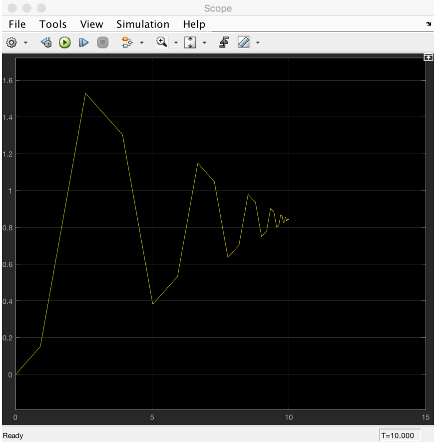
A Simulink Model



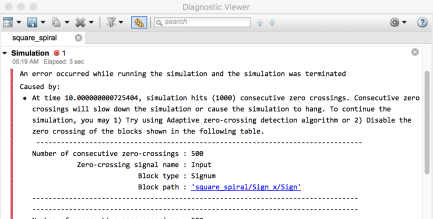
Stops at t=10



X Y Plot
1.5
1
0.5
0
-0.5
-1
-1.5
Y Axis
-1.5
-1
-0.5
0
0.5
1
1.5
X Axis



Scope
File
Tools
View
Simulation
Help
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
Ready
T=10.000



Diagnostic Viewer
search
square_spiral
Simulation 1
05:19 AM Elapsed: 3 sec
An error occurred while running the simulation and the simulation was terminated
Caused by:
At time 10.000000000725404, simulation hits (1000) consecutive zero crossings. Consecutive zero crossings will slow down the simulation or cause the simulation to hang. To continue the simulation, you may 1) Try using Adaptive zero-crossing detection algorithm or 2) Disable the zero crossing of the blocks shown in the following table.
Number of consecutive zero-crossings : 500
Zero-crossing signal name : Input
Block type : Signum
Block path : 'square_spiral/Sign x/Sign'

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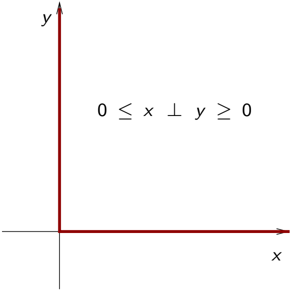
The Power of Complementarity Conditions

Scalar form: $0 \leq \lambda \perp \mu \geq 0$ with λ, μ scalar

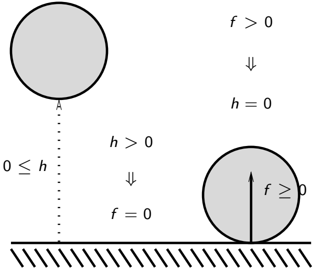
$$\lambda, \mu \geq 0 \text{ and } \lambda\mu = 0$$

General form: $\vec{\lambda} = \lambda_1 \ldots \lambda_n$ and $\vec{\mu} = \mu_1 \ldots \mu_n$
$0 \leq \vec{\lambda} \perp \vec{\mu} \geq 0$ means

$$\forall i = 1 \ldots n,\ 0 \leq \lambda_i \perp \mu_i \geq 0$$

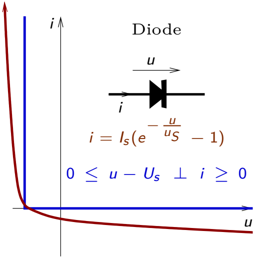


Multibody mechanics:



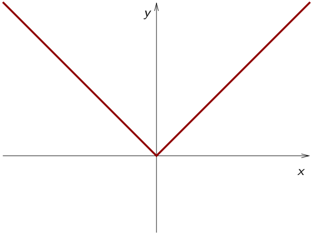
$$0 \leq h \perp f \geq 0$$

Electronics:



The Power of Complementarity Problems

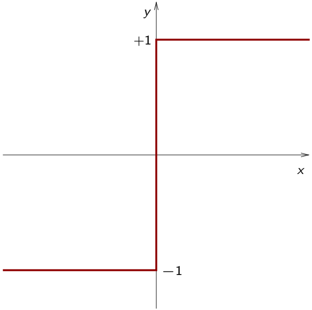
Example: $y = \textbf{abs}(x)$



$$\left\{ \begin{array}{rcl} y & = & u + v \\ x & = & u - v \\ 0 \leq u & \perp & v \geq 0 \end{array} \right.$$

$$\partial f(x) = \{ y \,|\, \forall z, f(z) - f(x) \geq y(z - x) \}$$

Example: $y = \textbf{sgn}(x)$



$$\left\{ \begin{array}{c} x \;=\; u - v \\ 0 \leq u \;\perp\; v \geq 0 \\ 0 \leq u \;\perp\; 1 - y \geq 0 \\ 0 \leq v \;\perp\; 1 + y \geq 0 \end{array} \right.$$

$$\textbf{sgn}(x) = \partial\textbf{abs}(x)$$

Linear Complementarity Problems & Systems

Definition: Linear Complementarity Problem (LCP)

$$\begin{cases} \lambda = M\mu + q \\ 0 \leq \lambda \perp \mu \geq 0 \end{cases}$$

Theorem: A LCP admits a unique solution for all q iff M is a P-matrix

Definition: M is a P-matrix iff all its principal minors $\det([M]_{i,i})$ are strictly positive

Definition: Linear Complementarity System = Linear ODE + LCP

$$\dot{q} = Aq + B\lambda$$

Zero-crossing as a complementarity system

$$\left\{\begin{array}{rcll} y(t_0) & = & 0 & \\ x & = & u - v & \\ 0 \le u & \perp & v \ge 0 & \\ 0 \le y' & \perp & v \ge 0 & \text{// As long as } x < 0,\ y \text{ is constant} = 0 \\ 0 \le 1 - y & \perp & u + y \ge 0 & \text{// As soon as } x > 0,\ y = 1 \text{ and stays there} \end{array}\right.$$

Remark: nondeterministic if x reaches and stays at 0

What is an event? switching from saturation of $u \geq 0$ to saturation of $v \geq 0$ in $0 \leq u \perp v \geq 0$

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Simulink Library Blocks: the Easy Cases

Coulomb and Viscous Friction:

$$\begin{cases} \quad y &= \quad Gx + Fs \\ \quad x &= \quad u - v \\ 0 \leq 1 - s \perp u \geq 0 \\ 0 \leq 1 + s \perp v \geq 0 \end{cases}$$

Dead Zone:

$$\begin{cases} y &= \quad k - v \\ x &= \quad u - v + L \\ x &= \quad k - l + U \\ 0 \leq u \perp v \geq 0 \\ 0 \leq k \perp l \geq 0 \end{cases}$$

More Simulink Library Blocks

Saturation Integrator:

$$\begin{cases} \dot{y} & = & x + k - l \\ 0 \leq y - L & \perp & k \geq 0 \\ 0 \leq U - y & \perp & l \geq 0 \end{cases}$$

Backlash:

$$\begin{cases} \dot{y} & = & k - l \\ 0 \leq y - x + \frac{B}{2} & \perp & k \geq 0 \\ 0 \leq x - y + \frac{B}{2} & \perp & l \geq 0 \end{cases}$$